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THE EFFECT OF SURFACE EMISSIVITY UPON INFRARED GASEOUS RADIATION*

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NOMENCLATURE

A, total band absorptance [cm^{-1}];
$\bar{A}$, dimensionless band absorptance, A/A_0;
A_0, band width parameter [cm^{-1}];
B_ω, spectral surface radiosity [$W/cm^2/cm^{-1}$];
C_0^2, correlation parameter [$atm^{-1}\ cm^{-1}$];
e_ω, Planck's function [$W/cm^2/cm^{-1}$];
$e_{1\omega}$, Planck's function evaluated at temperature T_1;
$e_{\omega c}$, Plancks' function evaluated at band center;
L, distance between plates [cm];
P, pressure [atm];
q_R, total radiation heat flux [W/cm^2];
$q_{R\omega}$, spectral radiation heat flux [$W/cm^2/cm^{-1}$];
Q, heat source or sink [W/cm^3];
T, temperature [°K];
T_1, wall temperature;
u, dimensionless coordinate, C_0^2Py;
u_0, dimensionless path length, C_0^2PL;
y, physical coordinate [cm];

Greek symbols

ε, surface emissivity;
κ_ω, spectral absorption coefficient [cm^{-1}];
ω, wave number [cm^{-1}].

INTRODUCTION

IN [1] AND [2] the authors have treated i.r. radiative heat transfer in nongray gases under the condition that the bounding surfaces are black. The purpose of the present note is to investigate the effect of nonblack surfaces upon i.r. radiative transfer. Specifically, this constitutes an extension of [1]. The illustrative physical model consists of an absorbing–emitting gas bounded by two infinite parallel plates, and there is a uniform heat source per unit volume, Q, within the gas. It is assumed that the plate surfaces are gray and diffuse and have the same temperature T_1 and emissivity ε. Furthermore, restriction is made to diatomic gases, such that, neglecting overtone bands, only a single fundamental vibration–rotation band is considered. Negligible thermal conduction and small temperature differences (temperature-independent absorption coefficient) are additionally assumed. With the exception that nonblack surfaces are considered, this is exactly the problem treated in [1].

ANALYSIS

Employing the exponential kernel approximation, $E_2(t) \simeq \frac{3}{4}\exp(-3t/2)$, the equation expressing the spectral radiation heat flux within the gas [3] may be written as

$$q_{R\omega} = \frac{3}{2}\int_0^y [e_\omega(z) - e_{1\omega}]\,\kappa_\omega \exp\left[-\frac{3\kappa_\omega}{2}(y - z)\right] dz - \frac{3}{2}\int_y^L [e_\omega(z) - e_{1\omega}]\,\kappa_\omega \exp\left[-\frac{3\kappa_\omega}{2}(z - y)\right] dz + (B_\omega - e_{1\omega})\left\{\exp\left[-\frac{3\kappa_\omega}{2}y\right] - \exp\left[-\frac{3\kappa_\omega}{2}(L - y)\right]\right\}. \tag{1}$$

Furthermore, making the same approximation for $E_2(t)$ in the equation describing the surface radiosity B_ω [3], one has

$$B_\omega - e_{1\omega} = \frac{\frac{3}{2}(1 - \varepsilon)\int_0^L [e_\omega(z) - e_{1\omega}]\,\kappa_\omega \exp\left(-\frac{3\kappa_\omega}{2}z\right) dz}{1 - (1 - \varepsilon)\exp\left(-\frac{3\kappa_\omega}{2}L\right)}$$

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Following Gille and Goody [4], the denomenator in equation (2) will be expanded, such that equation (2) becomes

$$B_\omega - e_{1\omega} = \frac{3}{2}(1-\varepsilon)\int_0^L [e_\omega(z) - e_{1\omega}]\sum_{n=0}^{\infty}(1-\varepsilon)^n \kappa_\omega \times \exp\left[-\frac{3\kappa_\omega}{2}(z+nL)\right]\mathrm{d}z. \tag{3}$$

Since the total radiation heat flux is given by

$$q_R = \int_{\Delta\omega} q_{R\omega}\,\mathrm{d}\omega \tag{4}$$

where $\Delta\omega$ indicates integration over the single band, then combining equations (1), (3) and (4), and following the identical procedure outlined in [1], yields

$$q_R = \tfrac{3}{2}\int_0^u [e_{\omega_c}(u') - e_{1\omega_c}]\,A'[\tfrac{3}{2}(u-u')]\,\mathrm{d}u'$$

$$-\tfrac{3}{2}\int_u^{u_0} [e_{\omega_c}(u') - e_{1\omega_c}]\,A'[\tfrac{3}{2}(u'-u)]\,\mathrm{d}u'$$

$$+\tfrac{3}{2}\int_0 [e_{\omega_c}(u') - e_{1\omega_c}]\sum_{n=0}^{\infty}(1-\varepsilon)^{n+1}\{A'[\tfrac{3}{2}u' + u + nu_0)] - A'[\tfrac{3}{2}(u' - u + u_0 + nu_0)]\}\,\mathrm{d}u' \tag{5}$$

where $A'(u)$ denotes the derivative of the total band absorptance with respect to u.

Now, from conservation of energy, $\mathrm{d}q_R/\mathrm{d}y = Q$, and

$$q_R = \frac{QL}{2}\left(2\frac{y}{L} - 1\right) \tag{6}$$

such that from equations (5) and (6), the integral equation describing the temperature profile within the gas is

$$\frac{u_0}{3}\left(2\frac{u}{u_0} - 1\right) = \int_0^u \phi(u')\,\bar{A}'[\tfrac{3}{2}(u-u')]\,\mathrm{d}u'$$

$$-\int_u^{u_0} \phi(u')\,\bar{A}'[\tfrac{3}{2}(u'-u)]\,\mathrm{d}u'$$

$$+\int_0^{u_0} \phi(u')\sum_{n=0}^{\infty}(1-\varepsilon)^{n+1}\{\bar{A}'[\tfrac{3}{2}(u' + u + nu_0)] - \bar{A}'[\tfrac{3}{2}(u' - u + u_0 + nu_0)]\}\,\mathrm{d}u' \tag{7}$$

where

$$\phi = \frac{e_{\omega_c}(T) - e_{\omega_c}(T_1)}{Q/PA_0C_0^2}.$$

For $\varepsilon = 1$ this reduces directly to the equation for black surfaces given in [1]. Numerical solutions of equation (7) have been obtained employing the band absorptance correlation of Tien and Lowder [5], and for a more complete discussion the reader is referred to [1].

Before presenting the numerical results, it will be of interest to investigate the two limiting solutions for $u_0 \ll 1$ and $u_0 \gg 1$. The limiting solution of equation (7) for the optically thin limit ($u_0 \ll 1$) is obtained by letting $\bar{A}'(u) = 1$, and from equation (7)

$$\phi(u) = \tfrac{1}{3} \tag{8}$$

which is identical to the result given in [1]. This invariance of surface emissivity upon gas temperature is also observed for a gray gas under optically thin conditions [6]. To explain this, recall that under optically thin conditions the surface radiosity is evaluated as if the gas were completely transparent [3], and since this corresponds to an isothermal enclosure for the present problem the surface radiosity is equal to black body radiation irrespective of the value of the surface emissivity.

Consider next the limiting form of equation (7) for large path lengths ($u_0 \gg 1$). This limit, which differs substantially from the conventional Rosseland limit [1, 2], is obtained by letting $\bar{A}'(u) = 1/u$ in equation (7)*, with the result

$$\xi - \tfrac{1}{2} = \int_0^1 \frac{\phi(\xi')}{u_0}\frac{\mathrm{d}\xi'}{\xi - \xi'} + \int_0^1 \frac{\phi(\xi')}{u_0}\sum_{n=0}^{\infty}\left\{\frac{(1-\varepsilon)^{n+1}}{\xi' + \xi + n} - \frac{(1-\varepsilon)^{n+1}}{\xi' - \xi + (n+1)}\right\}\mathrm{d}\xi' \tag{9}$$

where $\xi = y/L = u/u_0$. This equation illustrates that, in the large path length limit, the temperature profile, expressed as $\phi(\xi)/u_0$, depends solely upon the parameter ε. Unlike the black surface case [2], equation (9) does not possess a simple closed form solution, and thus a numerical solution to equation (9) has been obtained. In particular, the center-line temperature may be expressed by

$$\phi(\xi = \tfrac{1}{2}) = \gamma(\varepsilon)\,u_0 \tag{10}$$

where, from the numerical solution, values for $\gamma(\varepsilon)$ are as follows: $\gamma(1{\cdot}0) = 0{\cdot}159$, $\gamma(0{\cdot}5) = 0{\cdot}195$, $\gamma(0{\cdot}3) = 0{\cdot}218$, $\gamma(0{\cdot}1) = 0{\cdot}252$.

RESULTS

Results for the dimensionless center-line temperature, expressed in terms of Planck's function evaluated at the

* This corresponds to the large path length expression for the total band absorptance [1, 2], $\bar{A} = \ln u$.

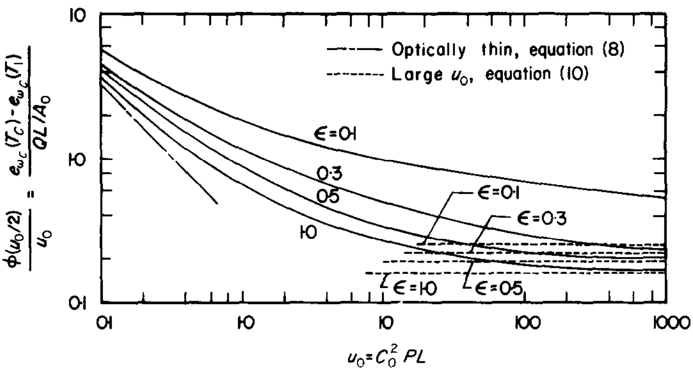

FIG. 1. Center-line temperature results for $\beta = 0{\cdot}1$.

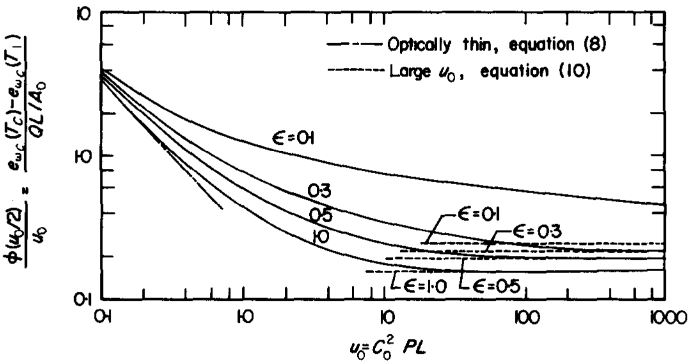

FIG. 2. Center-line temperature results for $\beta = \infty$.

band center, are illustrated in Figs. 1 and 2 for $\beta = 0{\cdot}1$ and $\beta = \infty$, respectively, where β is the line structure parameter and is proportional to the ratio of line width to line spacing [1]. As would be expected, a reduction in surface emissivity gives rise to a higher center-line temperature, since a lower surface emissivity corresponds to a reduction in the energy transfer capability between the gas and the surfaces.

Also shown in Figs. 1 and 2 are the two limiting solutions; that is, the optically thin limit described by equation (8), and the large path length limit corresponding to equation (10). It is seen that as ε is decreased, the ranges of applicability of the limiting solutions are appreciably reduced. In particular, for $\varepsilon = 0{\cdot}1$ an extremely large value of u_0 would be required in order to approach the large u_0 limit.

This departure from the limiting solutions with decreasing emissivity can also be shown to be characteristic of a gray gas. However, there is a specific difference between the present results for a nongray gas as opposed to those for a gray gas, and this involves the dependence on ε in the large path length limit. For a gray gas (or, for that matter, any gas with a nonvanishing absorption coefficient over the entire spectrum), the appropriate limit for large path lengths is the optically thick (or Rosseland) limit. In this limit the radiative transfer process is independent of ε [3], and thus the surface emissivity has no effect on the temperature profile within the gas for large path lengths. This is not the case, however, with reference to the present nongray results for infrared radiation.

The reason for the dependency on ε in the present large u_0 limit can readily be understood on physical grounds. In the large path length limit, radiation occurs solely in the band wings, and this involves a continuous transition from optically thick to optically thin radiation [1, 2]. Although as for a gray gas, neither optically thick nor optically thin radiation will be dependent upon ε, the surface emissivity does have an effect upon the radiative transfer occurring at intermediate optical thicknesses. In other words, in the large path length limit it is the radiation which is neither optically thick nor optically thin which produces the influence of ε upon the temperature profile within the gas.

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